Atomic Clocks and Variations of the Fine Structure Constant

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Abstract

We describe a new test for possible variations of the fine structure constant, α, by comparisons of rates between clocks based on diffe1 ent atoms. Clocks with different atomic number Z, e.g., H-maser, Cs and Hg^+ clocks, show sensitivity to variations in α via contributions of order $(Z\alpha)^2$ to the hyperfine clock transition. Recent H-maser vs. Hg^+ clock comparison data improves laboratory limits on a time variation by 100-fold to give $\dot{\alpha}/\alpha \leq 3.7\text{x}10^{-14}$/yr. This is the only laboratory test to preclude a cosmological variation of α of the sort predicted by Dirac's large number hypothesis.

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Since Dirac's large number hypothesis (LNH)[1], the search for a time variation of the fundamental constants has been the subject of much work[2]. Dirac noticed that the ratio of the electrostatic to gravitational forces between an electron and proton (-2x1039) was close to the age of the universe expressed in units of the light transit time across the classical electron radius, $R_e/c = e^2/(m_e c^3)$. He conjectured that these two very large quantities were proportional, hence, the ratio $e^2/(Gm_p m_e)$ would vary linearly with time. A fractional change $\delta G/G \approx -5 \times 10^{-11}$/year would result assuming a universe 2x1010 years old. Teller and other authors [2,3] have postulated a relationship for the fine structure constant α^{-1} -- $\log[hc/(Gm_e^2)]$ where $[hc/(Gm_e^2)]^{1/2}$-(electron Compton wavelength) /(Planck Length). Taken with the Dirac hypothesis of a time varying G, α may vary $\delta\alpha/\alpha$ - $\alpha(\delta G/G) \sim 3.6 \times 10^{-13}$/yr.

Several analyses of paleontological, geophysical and astronomical data were made apparently ruling out this strong variation hypothesis [2] though there have been conflicting claims for a measured variation of the gravitational constant[4]. The paleontological arguments were based upon the realization that even a small departure of the gravitational constant ,G, from the present day value would make the earth inhospitable to life. Arguments of this sort have arisen largely as a response to Dirac's LNH and have led to the development of the Anthropic Cosmological Principle (ACP). According to this princple[5], the large number ratio (LNR) values are not a consequence of the above proportionality postulated by Dirac but rather, the present day LNR values are one of a relatively small subset (of all possible LNR values) which will lead to the development of observers, i.e., physicists, astronomers, etc.

The experimental evidence against the LNH is divided into what might be called cosmological and modern measurements. For example, the most stringent limits on the electromagnetic coupling α follow from an analysis of isotope ratios $^{149}Sm/^{147}Sm$ in the natural Uranium fission reaction that took place some 2x109 years ago at the present day site of the Oklo mine in Gabon, West Africa[2,6]. This ratio is 0.02 rather than 0.9 as in natural samarium because of the exposure to the neutron flux from the uranium fission. It is thus deduced that the neutron capture cross-section in ^{149}Sm has not changed significantly in 2x109 years from its present day value. Modelling the neutron flux during that fission reaction and the sensitivity of the cross-section to variations in α it is reported that $\dot{\alpha}/\alpha \leq 10^{-17}$/yr. This limits the integrated change in α over the cosmological time period of 2x 10^{9} yrs. In a similar way, astronomical measurements of multiple spectral lines (with different dependence on α and other atomic constants) from a common source with a large cosmological red shift, have been used to place limits on variations of α over cosmological time periods of $\alpha/\alpha \leq 4\text{x}10^{-12}$/yr [7].

Modern or laboratory measurements are based on clock comparisons with ultra-stable oscillators of different physical make-up such as the superconducting cavity oscillator vs. Cesium hyperfine clock transition [8] or the Mg fine structure transition vs. the Cesium hyperfine clock transition[9]. Unlike the results inferred from phenomena taking place over cosmological time scales, the clock comparisons are repeatable and are of duration weeks to years. They rely on the ultra-high stability of the atomic standards and currently set limits competitive with all but the Oklo interpretation. The modern clock comparisons are really complementary to the cosmological determinations because they place a limit on a present day variation of α [10]. This is important because the standard cosmological world picture[11] predicts a time dependent expansion rate for the universe which may, according to Dirac's LNH, lead to a corresponding acceleration in any cosmological time variation of the constants. The present expansion rate is given by Hubble's constant and is different from the expansion rate -109 years ago depending on the size of the acceleration parameter[11].

Although the ACP may remove the the original motivation for suspecting a variation of the constants, it does not preclude such variation. Whether the constants vary will be determined by experiment and analysis. The present paper describes a new method for determining limits on the variation of α by comparing rates for clocks based on atoms of different atomic number Z. The method is based on the increasing importance of relativistic contributions to the hyperfine energy splitting as atomic number Z increases in the group I alkali elements and alkali-like ions. The contribution is a function of αZ which grows faster than $(Z\alpha)^2$ for the heavier atoms and thus differs for hydrogen ($Z=1$), rubidium ($Z=37$), cesium ($Z=55$), and Hg^+ ($Z=80$). Any variation in α, whether a cosmological time variation or a spatial variation via a dependence of α on the gravitational potential[12], will force a variation in the relative clock rates between any pair of these four clocks.

We begin by comparing the theoretical expressions for the hyperfine splitting (hfs) in hydrogen and the alkali atoms and ions. All continuously operated microwave atomic frequency standards (H, Rb, Cs, and Hg^+) are based on transitions between ground state hyperfine levels which are determined by the interaction of a nuclear magnetic moment with the magnetic moment of an $S_{1/2}$ state valence electron. The hydrogen hfs is the simplest and to lowest order in α and m_e/m_p, the splitting used as the clock transition in the H-maser is $a_s = \frac{8}{3}\alpha^2 g_p \frac{m_e}{n m_p} R_\infty c$ where g_p is the proton g factor, m_e and m_p are the electron and proton masses, and $R_\infty c$ is the Rydberg constant in frequency units.

The theory of the hyperfine splitting in alkali atoms and ions is not so well developed as that for hydrogen but much work has been done and the theoretical expressions predict the splittings for the Cs and Hg^+ clock transition frequencies to the 1% level[13]. The full expression for the hyperfine interaction constant A_s [13, 14] is

$$A_s = \frac{8}{3}\alpha^2 g_I Z \frac{z^2}{n_*^3}\left(1 - \frac{d\Delta_n}{dn}\right) F_{rel}(\alpha Z)(1-\delta)(1-\epsilon)\frac{m_e}{m_p} R_\infty c$$

The transition frequency between the $I \pm 1/2$ states is $(I+1/2)A_s$, where I is the nuclear spin angular momentum quantum number.

This expression is composed of several factors. The value of the valence electron wavefunction at the nucleus, obtained by solving the non-relativistic Schrodinger equation, is given by the semi-empirical Fermi-Segrè formula[15] $\psi_n^2(0) = \frac{Zz^2}{\pi a_0^3 n_*^3}\left[1 - \frac{d\Delta_n}{dn}\right]$

where Z is the atomic number, z is the net charge of the remaining ion following the removal of the valence electron, n_* is the effective quantum number chosen to match the measured energy levels, E_{n*}, for the alkali atom according to the Rydberg formula $E_{n*} = -z^2 Ry/n_*^2$. $\Delta_n = n - n_*$ is the quantum defect for the n'th state. The term (1-δ) is the correction for the departure of the atomic potential from pure Coulomb as the electron enters the relatively large high Z nucleus with $\delta \approx 4\%$ for Cs and 12% for Hg [13]. (1-ε) is a similar correction for the finite size of the nuclear magnetic dipole moment with $\epsilon \approx 0.5\%$ for Cs and 3% for Hg [13].

The Casimir correction factor $F_{rel}(\alpha Z)$ [13, 14, 16] is obtained when the relativistic wave equation is solved to evaluate the electron wavefunction in the vicinity of the nucleus. For an $S_{1/2}$ state electron $F_{rel}(\alpha Z) = 3[\lambda(4\lambda^2 - 1)]^{-1}$ where $\lambda = [1 - (\alpha Z)^2]^{1/2}$ showing F_{rel} is a strong function of α for high Z nuclei. For $\alpha Z \ll 1$, $F_{rel} \approx 1 + 11(\alpha Z)^2/6$ but with heavier atoms this approximation breaks down since for Cs, $F_{rel} = 1.39$ and for Hg, $F_{rel} = 2.26$.

A time variation in α will therefore induce a change in the frequency of an H-maser relative to the frequency of a heavy atom hfs transition according to

$$\frac{d}{dt}\ln\left(\frac{A_{alkali}}{a_{hydrogen}}\right) = \alpha\frac{d}{d\alpha}\ln(F_{rel})\left(\frac{1}{\alpha}\frac{d\alpha}{dt}\right)$$

We have assumed the integers z, and Z remain constant. Supposing that α changes, there will be a corresponding change in the effective quantum number n_* since it is determined by the Rydberg levels of the valence electron. However, because n_*^2 - $E_n/(z^2Ry)$ - (1+higher order in $(z\alpha)^2$) its changes are small. The finite nuclear volume correction δ does contain terms of order $(\alpha Z)^2$ but its overall sensitivity to α is $\leq$ 10% of that of F_{rel} and is negligible.

The above ratio between hyperfine transitions in different atoms contains no electron to proton mass ratio and the nuclear g-factors enter as a ratio unlike the clock comparisons described in references [8] and [9]. The above equation is re-written

$$\alpha \frac{d}{d\alpha} \ln (F_{rel}) = (\alpha Z)^2 \frac{12 \lambda^2 - 1}{\lambda^2 (4 \lambda^2 - 1)} = L_d F_{rel}(Z)$$

where $\lambda = \sqrt{1 - (\alpha Z)^2}$. The sensitivity to α of the ratio of these clock frequencies with increasing atomic number Z is shown in Figure 1.

By analogy with a Dime particle, the ratio g_I/g_p (g values of a bound nucleon to a free nucleon) is relatively insensitive to α. The nuclear g factors are defined as a ratio of the measured nuclear magnetic moment to the nuclear magneton $(e\hbar)/(2m_pc)$ and are determined primarily by the strength of the strong interaction. For an electron bound to a nucleus of charge Z there is a relativistic mass contribution to the electron g-factor of order $(\alpha Z)^2$ [14]. By contrast, the strong force binding a nucleon in a nucleus 'saturates', i.e., remains relatively constant with increasing atomic number unlike the electromagnetic binding of an electron to a nucleus. We therefore assume there is no corresponding contribution to the nuclear g-factor ratio which grows with atomic number Z as strong as the $(\alpha Z)^2$ dependence of F_{rel}.

As above, for the comparison of two clocks, each based on a transition in different alkali atoms with $Z>1$, there will be a relative drift in rates

$$\frac{d}{dt}\ln\frac{A_{Alkali1}}{A_{Alkali2}} = (L_d F_{rel}(Z_1) - L_d F_{rel}(Z_2))\frac{1}{\alpha}\frac{d\alpha}{dt}$$

Table 1 shows the size of the sensitivity $L_d F_{rel}(Z_1)$-$L_d F_{rel}(Z_2)$ for various clock intercomparisons that might be used to detect a temporal variation in α (or spatial with d/dt replaced by d/dU where U is the solar gravitational potential[12]). A larger sensitivity would cause a larger clock rate difference given a non-zero value for $\dot{\alpha}/\alpha$. Alternatively, given a variation in α, the six distinct drift rates of Table 1 would predict a clear signature which would be useful in discriminating against systematic errors that might show up in any single int ercomparison. For example, the Cs vs. Hg^+ rate difference should be $1.4 \div 0.74 \approx 1.9$ times greater than the H-maser vs. Cs rate difference, etc.

Several clock comparisons have been made which can be used to search for a variation of α. Long term comparisons of Cs to H-maser clocks are carried out in the generation and maintainence of the worldwide atomic timescale (TAI). A recent comparison carried out over a one year period between two cavity auto-tuned active H-masers and the primary cesium standards, CS 1 and CS2, (at PTB in Braunschweig, Germany) showed a 1.5x 10^{-16}/day relative frequency drift[17]. Similar clock comparisons have been made at the US Naval Observatory[18] with comparable clock rate drifts. Since

$L_d F_{rel}(55)=0.74$ we find $\dot{\alpha}/\alpha \leq 1.5\times10^{-16}$/day $\div$ $0.74 = 7\times10^{-14}$/yr.

We have developed [19] an ultra-stable frequency standard based on Hg^+ ions confined to a linear ion trap, and have recently completed[20] a 140 day clock rate comparison [to be published] between it and a cavity tuned H-maser[21]. In that comparison, a limit of 2.1 (0.8)x 10^{-16} /day was established for the frequency drift between - these two long term stable clocks. The Allan deviation of this clock comparison is shown in Figure 2. This is a more sensitive test for α variations than the Cs vs. H-maser comparison since $L_d F_{rel}(80) = 2.2$ and establishes an upper bound $\dot{\alpha}/\alpha \leq$ 3.7x10-14 /yr.

This Hg^+ vs H-maser limit represents a 10 fold improvement over the recent limit[9] and rules out the LNH variation of α (-3 .6x 10^{-13}/yr) discussed in the introduction. It should be noted that this result is the *only present day laboratory* test with enough sensitivity to rule out such variations. The limits established in ref [9] on an α variation ($\leq$ 2.7x 10^{-13}/yr) were inferred from astrophysical limits placed on $\alpha^2 g_p m_e/m_p$ [7]over a time interval of almost 10'⁰ yrs.

The Hg^+ vs. H-maser results presented here represent a 100-fold improvement over the best laboratory limits ($\leq$ 4x 10-]2 /yr) established in the superconducting cavity vs. Cs frequency comparisons of ref [8]. This improvement follows from the very good long term stability of the atomic Hg^+ and H--maser clocks, with relative drift - 10^{-16}/ day, as compared to the superconducting cavity oscillator where instrumental drifts can lead to frequency drifts of a few parts in 10*4/day [8].

In summary, we have developed a new method for detecting variations of the fine structure constant, α, by examining relative drift rates between atomic clocks which are continuously monitored in time scales in several labs worldwide. We have searched for such drifts in a clock comparison between Hg^+ and H-maser clocks and improved modern day limits on an α variation by an order of magnitude. Further improvements will follow as laser cooled Cs and Hg^+ [22]microwave standards are developed, and comparisons of clock rates should establish *instantaneous or modern day* limits on any temporal variation of α as good as the best cosmological results that have integrated such changes for over 109 yrs. Finally, this method also shows that comparisons between Cs, Hg^+, Rb and H-maser clocks can be used to improve the search for violations of the Einstein Equivalence Principle that would result from a dependence of α on the gravitational potential[12].

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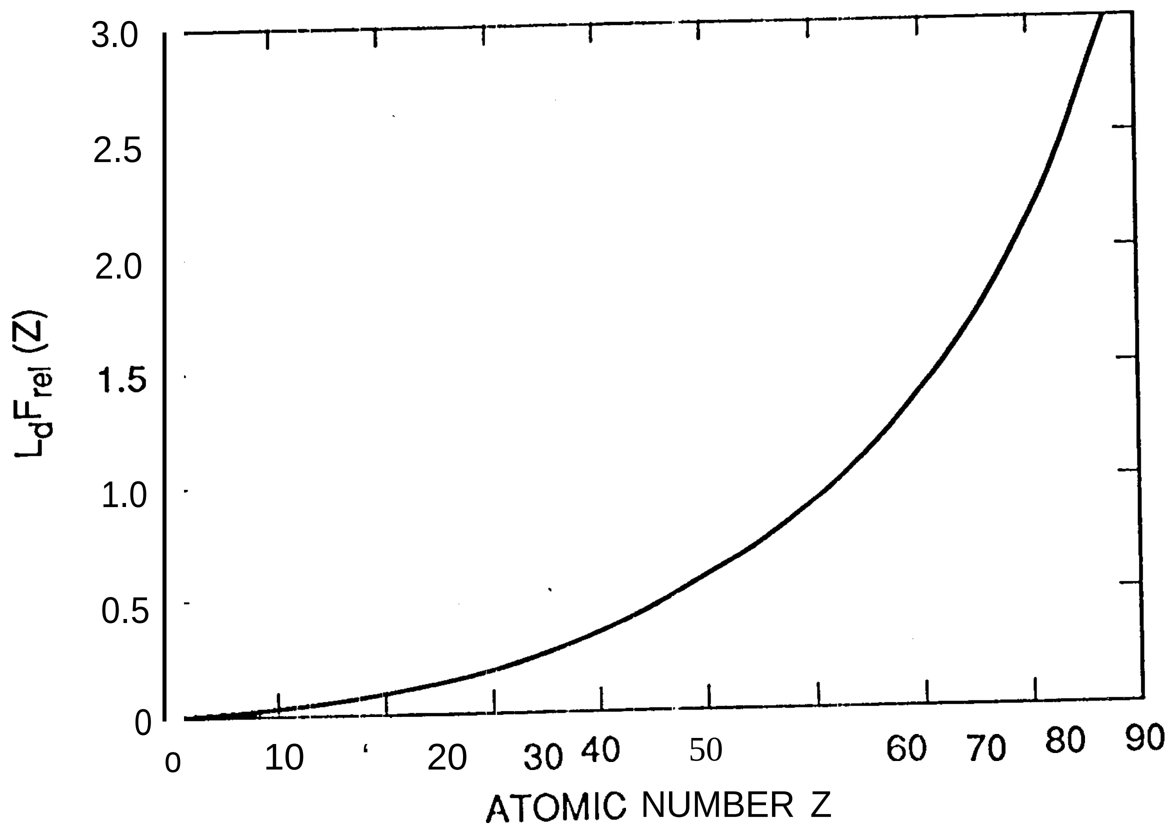
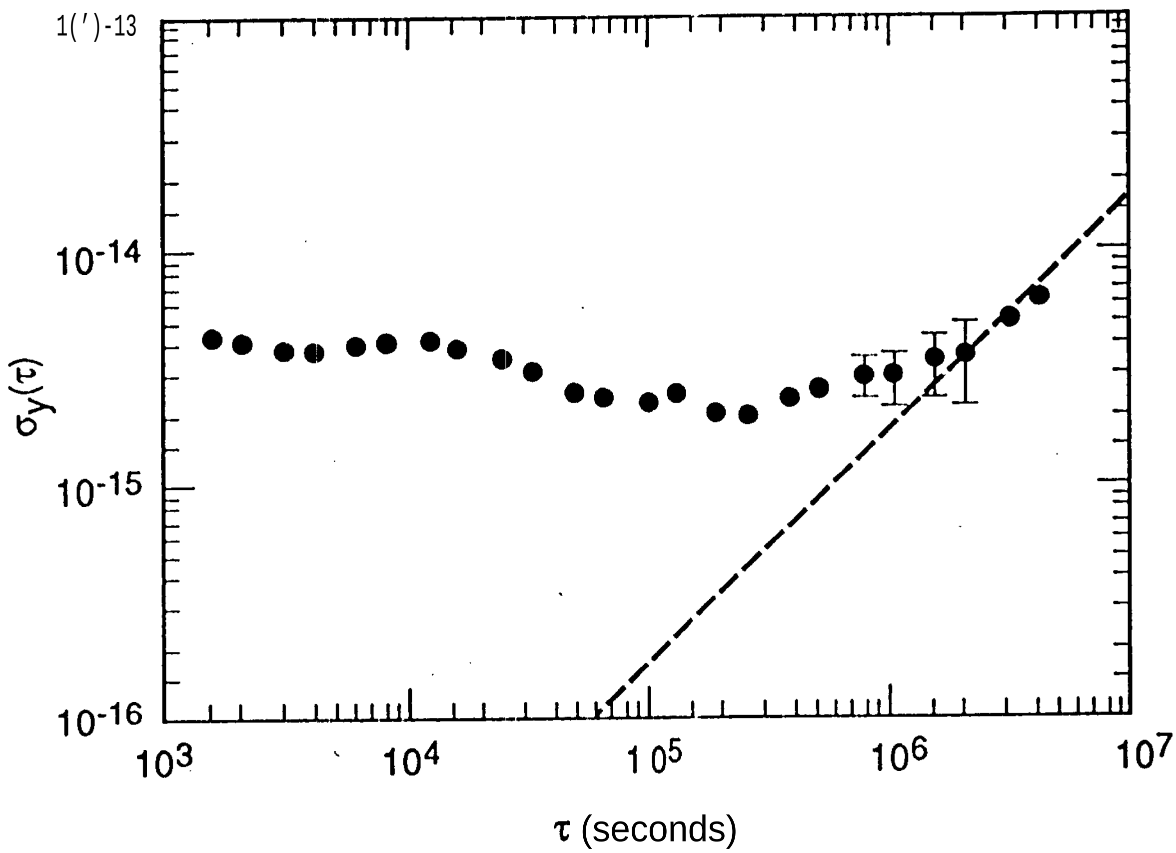
	H	Rb	Cs	Hg^+
H	0	0.3	0.74	2.2
Rb	-0.3	0	0.45	1.9
Cs	-0.74	-0.45	0	1.4
Hg^+	-2.2	-1.9	-1.4	0

Table 1: The sensitivity of various clock rate comparisons to a variable fine structure constant. The entry is $L_d F_{rel}(Z_1) - L_d F_{rel}(Z_2)$ and converts fractional changes in α to a drift in clock rates between the two given clocks. For example, if $\dot{\alpha}/\alpha = 10^{-14}$/yr, a frequency drift of 2.2x 10^{-14}/yr between an H-maser and an Hg^+ clock would result.

Figure 1: The function $L_d F_{rel}(Z)$ plotted against atomic number Z.

Figure 2: The measured Allan deviation for the 140 day H-maser vs. Hg^+ clock comparison.


3.0
2.5
2.0
1.5
1.0
0.5
0
$L_d F_{rel}(Z)$
0
10
20
30
40
50
60
70
80
90
ATOMIC NUMBER Z



1(′)-13
10-14
10-15
10-16
σy(τ)
10³
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1 0⁵
10⁶
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τ (seconds)